

VOLATILITY FORECASTING MODELS AND THEIR IMPLICATIONS FOR PORTFOLIO OPTIMIZATION: A COMPREHENSIVE REVIEW

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ABSTRACT

The concept of volatility is extremely important in measuring investment risk, pricing and portfolio performance. The growing number of financial crises, political uncertainties, various policies and technological innovations increase the complexity of markets. For this reason, accurate forecasting of volatility is required. Over the last four decades, there have been significant advances in the field of volatility forecasting from statistical and econometric models through machine learning and deep learning models. However, despite many achievements in the field, the current scientific literature lacks the synthesis of results regarding the comparative analysis of volatility forecasting approaches and its implications for portfolio optimization.

The goal of the paper is to conduct systematic and comprehensive review of the development of volatility forecasting models and its impact on portfolio optimization process. This paper presents the systematic literature review of the development of traditional statistical methods, GARCH family models, stochastic volatility models, machine learning and deep learning models and hybrid volatility forecasting models. Special attention is paid to the capability of each of these models to account for the stylized facts of financial time series: volatility clustering, leverage effect, persistence, long memory, nonlinearity and regimes.

This paper also studies the influence of volatility forecasts on portfolio optimization using different portfolio optimization methods: mean-variance optimization, minimum variance portfolios, risk parity portfolios and dynamic asset allocation portfolios. It is shown that more accurate volatility forecasts help improve risk measurement, asset allocation effectiveness, diversification and risk-adjusted returns of portfolios. Nevertheless, there are several problems associated with interpretability of the model, robustness of volatility forecasts in stressful periods, macroeconomic uncertainty and evaluation of volatility forecasting model performance from the economic perspective.

Based on systematic literature review, this paper develops the conceptual framework showing relationships between volatility forecasting models, risk measurement and portfolio optimization. Also, the research gaps in this field are outlined and the future research agenda including explainable artificial intelligence in volatility forecasting, hybrid econometric-machine learning approaches, regime-dependent volatility forecasting models, alternative data and uncertainty-aware portfolio optimization models is suggested.

Keywords: Volatility Forecasting, Portfolio Optimization, GARCH Models, Machine Learning, Deep Learning, Hybrid Forecasting Models, Dynamic Asset Allocation.

INTRODUCTION

Financial markets possess an inherent feature of uncertainty, continuous flow of information and interaction between economic agents, which leads to fluctuations in asset prices and returns. Fluctuations create volatility, defined as a measure of variation in asset returns through time and being one of the most important measures of financial risk. Volatility plays a key role in finance due to its impact on asset pricing, derivatives valuation, risk management and investment decisions. For investors and portfolio managers, accurate prediction of volatility becomes necessary to build an optimal portfolio and obtain superior risk-adjusted returns.

Over the past decades, the importance of volatility forecasts has significantly grown owing to the increasing complexity of financial systems, which experienced numerous situations of extreme uncertainty such as the Global Financial Crisis of 2008, the European sovereign debt crisis, the COVID-19 pandemic, geopolitical issues, supply chain disruptions, and monetary tightening policies of central banks. These situations led to turbulence in the financial market and revealed some limitations of traditional risk management tools, which usually presuppose relatively stable market conditions. At times of uncertainty, the distribution of asset returns demonstrates features such as volatility persistence, structural breaks, regime shifts, non-linear dependency, which makes the process of measuring financial risks and building optimal portfolios very difficult.

That is why volatility forecasting has become one of the most researched topics in financial economics and econometrics. The reasons for forecasting volatility stem from the fact that the volatility of financial markets varies over time and possesses certain stylized facts. First of all, financial volatility exhibits the phenomenon of volatility clustering, when high volatility is followed by high volatility and vice versa; secondly, there are leverage effects, when the negative shocks increase the volatility more compared to positive shocks; thirdly, financial volatility has long memory property, i.e. its shock effects persist through time; fourthly, time-varying correlation exists between financial assets. The presence of stylized facts means that the history of financial volatility contains valuable information to predict market uncertainty in the future.

Nevertheless, there were significant breakthroughs in literature related to volatility forecasting over the last four decades. Initially, simple statistics such as historical volatility, moving average, exponentially weighted moving average, and other techniques which enabled us to estimate market risks depending on past behavior were used. Even though those techniques were convenient for calculations, they were insufficiently powerful enough to capture complex dynamics of financial time series. One of the important breakthroughs in the field of volatility forecasting was the development of Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle (1982). According to that model, volatility is conditionally heteroskedastic and may be estimated as a function of forecast errors made earlier. This model has been further improved by Bollerslev (1986) who has developed Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model.

The success of GARCH model led to the development of various extensions of the initial idea which incorporated peculiar features of financial time series. The models such as Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), Fractionally Integrated GARCH (FIGARCH), and Dynamic Conditional Correlation GARCH (DCC-GARCH) allowed modeling asymmetry in responses to market shocks, long memory properties of financial time series, and correlations between asset returns respectively. Moreover, stochastic volatility models have become alternative approaches since they consider volatility as a latent

process. Overall, all these econometric models made a substantial contribution to the development of the field and became widely used instruments for research purposes in finance.

New advances in computer engineering, increasing availability of financial information and application of artificial intelligence brought about a new stage in the evolution of volatility forecasting research. Machine learning algorithms such as Artificial Neural Network, Support Vector Regression, Random Forest and boosting techniques proved to be efficient instruments in capturing complicated non-linear dependencies between variables. Later, deep learning techniques including Recurrent Neural Networks, LSTM, GRU, and Transformer became widely used due to their ability to capture temporal dependencies in data. Moreover, hybrid approaches that combine machine learning algorithms with econometric models started being used actively. However, despite the progress made in methodology, there still exists an important level of fragmentation of literature related to volatility forecasting and several challenges which have not yet been overcome. First, many studies concentrate on the problem of forecasting accuracy without considering its economic implications. Second, the findings related to model performance differ greatly depending on factors like asset class, market conditions and forecast horizon. Third, the problems connected with interpretability of models, robustness under market stress and incorporating macroeconomic and geopolitical uncertainties into forecasting remain under-researched. Fourth, relatively few studies focus on the relationship between volatility forecasting and portfolio optimization.

As was mentioned above, portfolio optimization needs accurate estimation of expected returns, variances and covariances of financial assets. Since volatility affects the risks of portfolios, incorrect estimates of these indicators may lead to inefficient portfolio formation and asset allocation resulting in poor investment performance. However, the advancements in volatility forecasting can contribute to improvement of risk assessment, dynamic portfolio rebalancing, hedging strategies and risk-adjusted returns. In this way, the analysis of interactions between volatility forecasting techniques and portfolio optimization becomes highly relevant for both academics and practitioners.

The significant amount of scientific literature has been devoted to volatility forecasting and portfolio management separately; however, there are relatively few reviews of the existing scientific research related to their integration and connections. Most of these reviews discuss a particular volatility forecasting technique or a particular class of models without considering the effects of this research for portfolio construction and management. The development of artificial intelligence technologies, access to new data and significance of uncertainty-aware investment strategies require the synthesis of existing scientific knowledge.

In such circumstances, the goal of the current study is to perform a systematic and critical review of the existing literature concerning the issue of volatility forecasting and to analyze the implications of the forecasting techniques for portfolio optimization and investment management. The main objectives of this study include the following five points. First, it is needed to review the evolution of volatility forecasting methods from traditional statistical models to modern machine learning and deep learning approaches. Second, it is required to critically evaluate the advantages and disadvantages of different forecasting methods in terms of their capability to reflect the stylized facts of financial time series. Third, it is necessary to examine the effect of volatility forecasts on portfolio optimization, risk management and dynamic asset allocation. Fourth, it is essential to reveal the gaps in existing scientific literature. Finally, it is needed to create an integrated theoretical framework and provide

recommendations for further research.



Fig. 1: Conceptual Framework Linking Volatility Forecasting and Portfolio Optimization

The rest of the paper is structured as follows. Section 2 explores the theoretical background of volatility of financial markets and portfolio optimization. Section 3 provides a review of methods for volatility forecasting, ranging from statistical and econometric tools to machine learning, deep learning, and hybrid models. Section 4 analyzes the role of volatility forecasting for portfolio optimization. Section 5 pinpoints research gaps and suggests research directions for future investigation. Lastly, Section 6 concludes the paper and summarizes the main findings and implications.

This literature review makes four key contributions to the existing body of knowledge.

First, it provides a systematic review of the evolution of volatility forecasting methodologies from the application of traditional statistical methods to modern artificial intelligence approaches.

Second, it analyzes the pros and cons of different forecasting methodologies and their capabilities to replicate the empirical features of financial returns.

Third, it explores the implications of volatility forecasting for portfolio optimization.

Fourth, it establishes an integrative research framework and proposes a research agenda focusing on explainable artificial intelligence, alternative data utilization, and uncertainty-aware portfolio optimization.

THEORETICAL BACKGROUND AND EVOLUTION OF VOLATILITY FORECASTING

Financial Market Volatility

Volatility is considered one of the major concepts in financial economics which characterizes the degree of dispersion or variability of the asset's returns in some certain period of time. In the financial market volatility is associated with uncertainty and risk, being applied in order to quantify the deviations of asset prices from their average values. So, in case the level of volatility is high, it indicates great uncertainty and considerable variations in asset prices, while in case the level is low, it indicates a stable state of financial markets. Consequently, volatility became an integral part of financial decisions influencing such areas as asset pricing, derivatives pricing, investment portfolio construction and management, risk assessment.

In the sphere of investments volatility is perceived as the measure of risk because people do not like uncertainty and risk but prefer stability and demand higher returns in case the level of uncertainty is high. The relationship between risk and return is one of the most important

concepts in contemporary finance and the foundation of various asset pricing and portfolio allocation models. Given the direct influence of volatility on the measurement of the portfolio risk and optimal asset allocation, the correct estimation and forecasting of volatility are especially necessary.

There are several kinds of volatility mentioned in the financial literature. Historical volatility is measured by historical returns and shows the backward estimate of risk. Realized volatility is based on high frequency data about asset prices and is used to estimate real market movements in some certain periods. Implied volatility is derived from options' prices and reflects the market participants' expectations concerning uncertainty. And finally, conditional volatility is a time varying measure of risk and is estimated using econometric models. It is this kind of volatility which deserves particular attention because it considers the idea of risk variability depending on the past and current market situation.

Stylized Facts of Financial Returns

It has been proved empirically that financial assets exhibit certain statistical features different from the ones implied by the assumption of independent identically distributed observations. These statistical characteristics of financial assets were called stylized facts of financial returns. They became the basis for the need to develop sophisticated methods of volatility forecasting. The first one and the most extensively studied characteristic of financial assets is the presence of the phenomenon of volatility clustering, i.e., high volatility follows high volatility and vice versa. It means that this property allows predicting the future volatility using the past information to some degree.

Also, another feature of financial assets is the existence of the leverage effect meaning that the volatility changes in case of positive or negative shock do not coincide. It has been empirically proven that negative returns increase volatility higher than the positive returns of the same magnitude. It is especially true for the equity markets and requires developing nonlinear models of volatility.

Besides, the distribution of financial returns exhibits the characteristics of fat tails and excess kurtosis, i.e., extremely high or low values appear more often than would be expected for normal distribution. It means that financial crises occur more often than in case of classical statistical models. Additionally, financial returns demonstrate such characteristics as long memory and persistence of the effects caused by volatility shocks. Structural breaks, regime shifts, and nonlinearities make up for the complicated behavior of the financial markets.

Thus, the characteristics described above imply that the volatility of financial markets is dynamic, persistent, and nonlinear. Therefore, forecasting techniques should consider these stylized facts.

Table I summarizes the major stylized characteristics of financial return series and their implications for volatility forecasting.

TABLE I: Stylized Facts of Financial Returns and Their Implications

Stylized Fact	Description	Implications for Forecasting
Volatility Clustering	Periods of high volatility are followed by high volatility, and periods of low volatility are followed by low volatility.	Requires models capable of capturing time-varying conditional variances such as ARCH and GARCH models.
Leverage Effect	Negative shocks generally increase volatility more than positive shocks of similar magnitude.	Motivates asymmetric models such as EGARCH and TGARCH.

Fat Tails and Excess Kurtosis	Extreme observations occur more frequently than predicted by the normal distribution.	Requires robust forecasting techniques capable of modeling tail risk and extreme events.
Long-Memory Behaviour	Volatility shocks decay slowly and may persist for prolonged periods.	Motivates long-memory specifications such as FIGARCH models.
Time-Varying Correlations	Relationships among asset returns evolve continuously over time.	Requires multivariate models such as DCC-GARCH for portfolio applications.
Structural Breaks and Regime Shifts	Market dynamics change significantly during crises and major economic	Encourages regime-switching and adaptive forecasting frameworks.
	events.	

Volatility Forecasting and Risk Management

By the term "volatility forecasting," we mean the procedure that involves the use of the historical data and calculations for the prediction of future risks. The primary purpose of volatility forecasting is the creation of an accurate forecast of future risks which could be used in making financial decisions.

Volatility predictions are used in financial economics for a variety of reasons. Firstly, volatility forecasts are used for option pricing and, thus, they are necessary in the derivative markets. Secondly, it is necessary to forecast volatility to calculate different risk measures including such concepts as Value at Risk, Expected Shortfall, stress testing and scenario analysis. Financial institutions forecast volatility to determine capital requirements and risk limits.

Moreover, in terms of investment, volatility forecasts are helpful in predicting future changes and making investments based on the forecasts. The financial market is characterized by the time-varying risk property and, therefore, static investment strategy becomes ineffective when the risk level is high.

Portfolio Optimization and the Role of Volatility Forecasts

Portfolio optimization refers to the process of allocating wealth to various investments in order to meet particular investment objectives and account for the risks involved. According to modern portfolio theory introduced by Markowitz (1952), the selection of portfolio should be made under consideration of expected returns as well as portfolio risk, which is assessed using either the variance or the standard deviation of the returns and, consequently, depends on the volatilities of assets and their correlation.

Thus, the accuracy of the volatility estimates becomes a crucial point for the efficiency of the process of portfolio optimization. Underestimating the value of volatility means the increase in risk and poor diversification, while the overestimation of the volatility leads to excessive conservatism and low yields. The use of proper volatility estimates improves the efficiency of risk assessment, asset allocation, and the benefits of portfolio diversification.

Moreover, there is another set of applications of volatility forecasts in recent portfolio management techniques such as minimum variance portfolios, risk parity strategies, and dynamic asset allocation, where portfolio weights are regularly updated according to changing market environment and risk perceptions.

Evolution of Volatility Forecasting Methodologies

The development of forecasting techniques is the reflection of the ever-growing

understanding that the volatility of financial markets changes and follows rather complicated patterns statistically. Simple forecasting techniques used in the beginning were based mostly on the usage of historical averages and simple statistics. This approach implied that market conditions are stable and that variances do not change. While the calculations based on simple statistics are relatively simple to carry out, this approach did not consider the fact that the dynamics of the volatility of financial markets should be modeled properly.

Further developments in financial econometrics resulted in the invention of models of conditional heteroskedasticity such as models of the families of ARCH and GARCH. Several additional models that allowed to consider the effects of asymmetry, long memory, and dynamic dependencies between financial instruments were developed later.

The development of computing technologies and the availability of large amounts of data for financial applications allowed using the methods of machine learning and deep learning in forecasting problems. Such techniques allow to use the complex and nonlinear relationships between different variables. Recently, hybrid models of forecasting combining econometrics and artificial intelligence started to be considered as one of the promising directions in forecasting. The described evolution shows how the research in volatility forecasting evolves continuously.

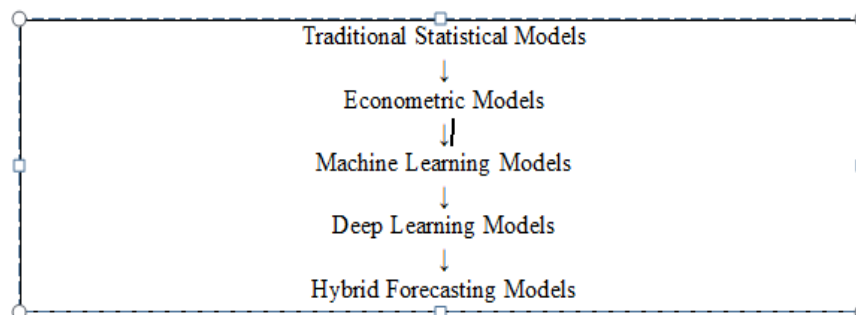


Fig. 2: Evolution of Volatility Forecasting Methodologies

REVIEW OF VOLATILITY FORECASTING MODELS

Forecasting volatility in financial markets has always been one of the most significant research topics in the field of financial economics, risk management, and portfolio theory. Volatility forecasting becomes crucial as the uncertainty regarding the future states of financial markets affects such areas as asset pricing, derivatives pricing, risk management, portfolio construction, and other financial decision-making issues. Given the modern world, where the probability of financial crisis is quite high, there is much uncertainty in the macroeconomic environment, and there is a high level of interconnectedness of global financial markets, the need to develop advanced volatility forecasting models becomes particularly important as they should be capable of taking into account the dynamical features of financial time series (Poon & Granger, 2003).

Volatility forecasting can be viewed as a challenging task as there are numerous stylized facts about financial return series that complicate the process of forecasting. The first one is volatility clustering meaning that high-volatility episodes follow each other. The second one is the leverage effect stating that a negative shock leads to an increased level of the volatility in comparison with a positive shock. The third fact is fat tails and excess kurtosis imply that extreme events occur more often than in the case of normal distribution. The last fact is the long memory implying that shocks to volatility are persistent for a relatively long period of time (Cont, 2001). All those stylized facts contradict to the assumption regarding the constancy of variance and normality of returns used in traditional financial models,

and it motivate researchers to create complicated forecasting models.

The development of the volatility forecasting models is a multistage process. At the beginning, simple statistical methods based on the prices observed in the past and assuming the stability of financial markets were employed. Although those methods are highly efficient from the computational perspective, they fail to account for the dynamic and persistence properties of financial market volatility. Then, the introduction of the models of conditional heteroskedasticity made the revolution in the field of financial econometrics as they accounted for the time-varying variance and stochastic nature of the volatility (Engle, 1982; Bollerslev, 1986). There were many modifications of those models developed to consider asymmetry, persistence, long memory, and time-varying dependencies between financial assets.

Nowadays, given the development of computational power and the availability of data and artificial intelligence, the application of machine learning and deep learning methods to volatility forecasting has become widespread. Those approaches allow estimating highly nonlinear relationships and extracting information from big and complex datasets without any assumptions regarding the distribution of variables (Gu, Kelly, & Xiu, 2020). Furthermore, there were hybrid approaches combining econometric and machine learning approaches suggested as potential solutions for improving forecasting accuracy.

Given the variety of approaches to volatility forecasting and the lack of perfect models, it is important to perform the critical analysis of the theoretical background and empirical performance of those approaches along with their strengths and weaknesses. Thus, this section will discuss various types of volatility forecasting models including statistical models, econometric models, machine learning, deep learning, and hybrid forecasting approaches.

Traditional Statistical Models

Traditional models for forecasting volatility are those that focus more on past data and apply simpler statistical methods. This kind of methodology assumes that there exists some valuable information related to the future volatility from the movements of past prices, and therefore, different averaging techniques are used to forecast volatility. Due to their ease of application, they have been widely adopted in finance for risk evaluation and portfolio management purposes. However, because of their inability to capture changing risks in the markets due to assumptions of stability, these models are often not very effective in predicting volatility in dynamic financial situations (Poon & Granger, 2003).

Historical Volatility

Historical Volatility (HV) may be considered as one of the oldest and most widely used risk measures of financial markets. Historically, the measure has been usually calculated as the standard deviation of asset return over some observation period and represents a back looking estimate of market uncertainty. The assumption of the model of historical volatility is the presence of information about future risks contained in the historical return volatility.

The main advantages of historical volatility model are related to its simplicity, convenience and easy computations. Being very simple, the model does not require any complex process of parameters estimations, which makes it possible to use this model for assets and financial markets with different characteristics. Therefore, historical volatility model is still very popular among the market participants as a benchmark risk measure and first signal about changes in market conditions.

However, at the same time, there exist several important limitations of this model. First, the historical volatility model assigns the same weight to all previous historical values and

assumes the constancy of market volatility. The assumptions mentioned are often not true as the financial markets are usually subject to volatility persistence, economic changes and structural breaks (Cont, 2001). Moreover, historical volatility models cannot consider such features of financial data as volatility clustering, leverage effect or non-linearity.

Exponentially Weighted Moving Average (EWMA)

The Exponentially Weighted Moving Average (EWMA) model is another approach that was invented to overcome the shortcomings of historical volatility estimation. Unlike historical volatility, the EWMA assigns exponentially decreasing weights to earlier observations and prefers to use more recent market data. The reason for that is the fact that the recent shocks are better predictors of future risk in the market than the historical observations.

There are several advantages of the EWMA model. Firstly, it is quite simple and effective from the perspective of computations. Secondly, being more sensitive to the recent observations, it adapts faster to the changing conditions and reacts quicker when the volatility becomes high. It is the simplicity and forecasting ability that make the EWMA model so popular among financial institutions and market risk measurement frameworks. It even became part of the RiskMetrics system developed by J.P. Morgan (J.P. Morgan, 1996).

At the same time, there is a set of drawbacks of the EWMA model. Firstly, it assumes constant decay rate, which means that the model cannot detect complicated changes in the market environment and volatility regime. Another drawback of the model is its inability to account for asymmetry, long memory and non-linear relationships between variables. Therefore, even though it outperforms simple historical volatility, the forecasting ability of the model cannot compete with the more complicated approaches (Poon & Granger, 2003).

Overall, the shortcomings of the traditional statistical approach have led to development of more sophisticated models.

Econometric Volatility Models

Econometric volatility models have been amongst the greatest achievements made in financial forecasting studies. Different from statistical methods, econometric models assume that financial market volatility does not remain constant over time and varies systematically in response to past information. Econometric volatility models have been developed since empirically, financial return series are characterized by volatility clustering, persistence, asymmetry, and non-normality, all of which cannot be explained by means of the constant variance models (Engle, 1982; Cont, 2001).

In conditional heteroskedasticity models, the level of volatility is a dynamic process dependent on past shocks to the market and past volatility dynamics. Due to the ability to model time-varying conditional variances, these models became important tools used in financial econometrics, derivative pricing, portfolio management, and risk measurement (Bollerslev, Chou, & Kroner, 1992).

For the past four decades, the field of econometric volatility has developed considerably, evolving from the Autoregressive Conditional Heteroskedasticity (ARCH) model into more advanced models allowing modeling asymmetries, long memory properties, and interrelations between financial instruments. Altogether, these models greatly helped to understand financial markets and became very popular in the field of volatility forecasting.

Autoregressive Conditional Heteroskedasticity (ARCH)

Autoregressive Conditional Heteroskedasticity (ARCH), which was developed by Engle in 1982, represents another fundamental discovery in financial econometrics. The standard

approach to dealing with time series implies that variances of the error terms in the regression models remain homoscedastic throughout the entire period of analysis. However, the empirical evidence suggests that financial returns show episodes of high and low volatility that last for some period and thus violate the condition of homoskedasticity in financial markets.

The ARCH model introduces the idea that conditional variance of a time series depends on the squared forecast errors of the previous periods. Therefore, under this model, large shocks are followed by large shocks and smaller shocks lead to the periods of stability, which allows the model to incorporate one of the stylized facts of financial returns – volatility clustering (Cont, 2001).

The advent of the ARCH model has changed the approach to analyzing financial processes dramatically because it has shown that volatility could be predicted in some way and is a process rather than a static factor. Many studies have used this method of modeling stock markets, exchange rates, prices of commodities and interest rates and found significant improvements in risk estimation compared to the approaches that assume constant variances (Bollerslev et al., 1992).

However, there are certain drawbacks to the ARCH model. The main problem of the model is the necessity to use many parameters for describing volatility because of the large number of lags of the error terms necessary to describe the process. Besides, the model fails to explain asymmetries in reaction to positive and negative shocks and is unable to account for the high degree of persistence of volatility. Nevertheless, this problem prompted researchers to develop more flexible and parsimonious models of heteroskedasticity.

Nonetheless, the ARCH model represents one of the milestones in the field of financial econometrics because it laid the theoretical foundation of the future volatility forecasting techniques.

Generalized Autoregressive Conditional Heteroskedasticity (GARCH)

To eliminate the drawbacks related to the ARCH approach, Bollerslev (1986) developed the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) approach that became one of the most widely used and influential models in financial econometrics. The key feature of the GARCH approach is its capability to consider the influence of past conditional variances on future volatility estimates besides the effects of previous forecast errors. Thus, volatility becomes a function of recent market shocks and its past values, providing more efficient representation of financial risk dynamics.

The use of the GARCH framework is highly beneficial in capturing volatility clustering and persistence – features that are common for all financial markets. As periods of high uncertainty are followed by other periods of increased volatility, introduction of the lags of the conditional variances substantially increases forecasting efficiency and leads to more realistic assessments of market risk (Poon & Granger, 2003).

There are several reasons for the popularity of the GARCH models. First, the model is successful in considering the time-dependent nature of financial volatility at the relatively low level of computational complexity. Second, it demonstrates superior forecasting performance in comparison with the simpler statistical models and proves its efficiency in equity, foreign exchange, commodity and fixed-income markets (Hansen & Lunde, 2005). Finally, the flexibility of the model made it highly applicable for the purposes of pricing derivatives, calculating Value-at-Risk and optimizing portfolios.

However, there are some disadvantages of the traditional GARCH approach. For instance, the

basic GARCH model implies the same impact of the positive and negative shocks on future volatility estimates. In practice, negative events lead to greater volatility changes than the positive information of equal value, which is commonly known as the leverage effect (Black, 1976). Moreover, GARCH models face certain problems related to structural breaks, nonlinearity and long memory in times of financial turmoil.

All these issues promoted the emergence of many extensions of the GARCH model such as Exponential GARCH (EGARCH), Threshold GARCH (TGARCH), Fractionally Integrated GARCH (FIGARCH) and Dynamic Conditional Correlation GARCH (DCC-GARCH) models.

Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH)

The Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) specification, suggested by Nelson (1991), was developed explicitly to address one of the major limitations of the classic GARCH specification – the inability of the former to capture the asymmetric influence of positive and negative innovations on the level of volatility. Empirical evidence suggests that bad news affects the volatility to a larger extent than the good news of equal magnitude (so-called “leverage effect” Black, 1976). Since GARCH models are symmetric, they do not capture this aspect of volatility dynamics and hence underestimates the risk during turbulent periods in the markets.

This shortcoming was addressed by modeling the logarithm of the conditional variance and hence allowing for various influences of positive and negative shocks on future levels of volatility. In modeling the volatility in the logarithmic form, EGARCH is not limited to certain parameter values and guarantees the positivity of conditional variances (Nelson, 1991).

The ability of EGARCH models to consider asymmetric influence on volatility makes them widely used in analyzing equity markets, exchange rates, commodity markets and risk management. Empirical research has shown that EGARCH models are superior to GARCH models in cases when financial markets exhibit leverage effects and asymmetric information transmission (Poon & Granger, 2003). For this reason, EGARCH models are particularly valuable for estimating Value at Risk, portfolios’ risk analysis and derivative pricing.

There are several limitations associated with EGARCH specification. It is computationally complex in higher order specifications and can be quite sensitive to distributional assumptions and methods of parameters estimation. Moreover, despite EGARCH models being rather effective in modeling asymmetries, they may fail to model long memory or regime shifts.

Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH/GJR-GARCH)

The TGARCH or GJR-GARCH model was developed by Glosten, Jagannathan, and Runkle in 1993 for further elaboration of the asymmetric volatility phenomenon in finance. The similarity of the TGARCH model to the EGARCH model is its possibility to consider the difference between the effects of the positive and negative shock on the future volatility.

The most significant advantage of the TGARCH approach in comparison with existing models is the inclusion of the threshold variable into the model to ensure that the effect of the negative innovation would be greater than the effect of the positive one of the equal magnitudes. This feature of the model can be especially useful in the analysis of the behavior of financial markets, since the bad news is always associated with pessimism, uncertainty, and risk-averse behavior of the investors leading to unusually rapid volatility rises.

TGARCH models proved to be very useful for modeling volatility of the equity market, and they were often used in research on such issues as financial crises, contagion, and risks transfer between markets. It was stated that threshold models usually perform better than the symmetric GARCH models in case of the high uncertainty level and fast changing investor sentiments (Brooks, 2019).

It is also possible to point at some weaknesses of the TGARCH model. Like other members of the GARCH family, this model can have some problems in estimation of the long memory phenomenon and multiple structural breaks. Additionally, the forecasting ability of the model can vary according to the market and investment horizon.

Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH)

While GARCH-type models effectively incorporate volatility clustering and persistence, empirical evidence shows that volatility of financial markets is long memory, meaning that volatility shocks have a long-lasting effect on market uncertainty. To correct this problem, Baillie, Bollerslev, and Mikkelsen (1996) suggested a new approach, called Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH).

By introducing a fractionally integrated approach in the conditional variance equation, FIGARCH allows for hyperbolic and not exponential decay of volatility shocks, thus providing a better depiction of reality since the effects of uncertainty created by economic, financial, or policy shocks can affect market conditions during prolonged periods of time.

The ability of FIGARCH models to represent the long-range dependence in the data has made it popular for applications in the equity market, currency market, and commodities market. Empirical evidence often finds better forecasting accuracy of FIGARCH models in environments with high persistence and market uncertainty (Baillie et al., 1996; Poon & Granger, 2003). However, the drawback of this approach is the computational burden it imposes and the need for large samples to estimate parameters reliably. Also, even though FIGARCH captures long memory well, it does not capture asymmetry and sudden shifts in regime of volatility.

Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH)

The Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH), developed by Engle (2002), can be considered a significant development in extending univariate volatility models into multivariate settings. Although classic GARCH techniques are mostly oriented on estimating individual volatilities of financial assets assuming fixed correlations between them, in practice, correlations appear to be time-variant and increase under stressful economic conditions.

In order to address this issue, DCC-GARCH technique provides one with the opportunity to evaluate time-variant volatilities and dynamic conditional correlations of multiple financial assets at once. In such a way, it offers a more comprehensive representation of market risks and interactions and appears quite useful in analyzing financial contagion, integration of international markets, and portfolio diversification.

The relevance of DCC-GARCH methodology in practical applications is particularly evident in the context of portfolio optimization and asset allocation issues as in addition to volatilities, risk of the portfolio is dependent on the correlations between its assets as well. Therefore, an adequate assessment of the time-variant correlations is essential for enhancing the effectiveness of portfolio diversification and optimizing portfolio performance. Numerous

researchers have already utilized DCC-GARCH in dynamic asset allocation, hedging strategy, and risk management and found it effective in providing more accurate risk assessments and higher portfolio efficiency (Engle, 2002; Silvennoinen & Teresita, 2009).

Despite the flexibility and applicability of the DCC-GARCH technique, however, with the increasing number of assets, it becomes rather complex computationally. Moreover, it is prone to being sensitive to some distributional and parameter considerations in highly turbulent markets with significant structural shifts and nonlinear dependencies.

Machine Learning Models

The rapid growth of computational technologies, the availability of big data in finance, and the complexity of financial markets led to an increase in the application of machine learning to the volatility forecasting problems. Contrary to econometric models based on some predefined functional form and distributional assumptions, machine learning is a data-driven approach that allows modeling highly non-linear relations between variables (Gu, Kelly, & Xiu, 2020). Machine learning models are applied in finance because of the assumption about the complexity of financial markets – the system consisting of adaptive and dynamic processes featuring non-linear dependencies, structural breaks, and a high dimensional information environment (Hastie, Tibshirani, & Friedman, 2009). Such features of financial markets cannot be captured by conventional econometric models, especially during the periods of turbulence in markets. Machine learning models allow overcoming these limitations through finding patterns in data and uncovering hidden dependencies in information. Some papers demonstrate that machine learning models perform better than conventional econometric models in terms of their prediction power, especially for the cases of using big data sets and numerous explanatory variables (Gu et al., 2020). For this reason, machine learning techniques have been widely used in such areas of finance as volatility forecasting, algorithmic trading, asset pricing, portfolio optimization, and financial risk management. Machine learning models possess certain benefits but at the same time they have certain drawbacks as well. First, most of the machine learning models work as black-box models, which means that they do not have economic interpretation of market volatility determinants. Second, machine learning models may be subject to overfitting and require significant amount of computing resources and parameter tuning. Therefore, while being extremely flexible and powerful forecasting tools, these approaches should be used for their interpretability as well as their prediction ability. This chapter reviews the most used machine learning models in volatility forecasting studies.

Artificial Neural Networks (ANN)

Another class of machine learning methods that was applied in financial forecasts is Artificial Neural Networks (ANNs). ANNs are inspired by the processes occurring in biological neural networks. They represent networks with connected processing units forming input, hidden, and output layers (Haykin, 2009).

The primary strength of ANN models is that they can approximate very complicated nonlinear functions without making restrictive assumptions about the process that generates data. Since financial markets are characterized by nonlinearity of relationships between economic and financial indicators, ANNs offer an adequate approach to forecast market volatility and risk.

There are many cases where the use of ANN models proved to be more efficient in predicting financial market volatility than the use of traditional econometric specifications, especially in situations where financial market behavior is characterized by nonlinearity (Zhang, Patuwo,

& Hu, 1998). Therefore, neural networks have been widely used for forecasting the movements on stock markets, exchange rates, prices of derivatives, and managing investment portfolios.

Despite their benefits, ANN models have many drawbacks. Estimation of ANN models often requires vast amounts of data and substantial computing resources. Furthermore, the models of such type usually demonstrate poor interpretability as the relationship is incorporated into many interconnected parameters.

Support Vector Regression (SVR)

Support Vector Regression (SVR) is a machine learning technique that extends support vector machines for use in regression and forecasting tasks. This approach is based on statistical learning theory, and its goal is to find the optimal regression function, which minimizes prediction errors and allows generalizing models (Vapnik, 1995). There are two reasons for using SVR in volatility forecasting. First, support vector regression enables capturing the nonlinear relationship through the application of kernel functions. In contrast to linear models, SVR models can detect complex relationships between financial variables and provide accurate predictions even when data samples are small.

Research proves that SVR models perform better than some econometric techniques in predicting the volatility of stock market and financial risk especially in nonlinear markets (Kim, 2003). Moreover, SVR models have relative high generalization properties, and they are less prone to overfitting than some neural networks.

However, the performance of SVR models in forecasting depends strongly on choosing the appropriate kernel functions and parameters. Additionally, the methodology may face some computational problems when applied to huge datasets and does not allow making an economic interpretation of variables affecting market volatility.

Random Forest

Random Forest is an ensemble learning technique developed by Breiman (2001), which generates several decision trees and makes predictions based on averaging their results to increase predictive power and stability. The technique makes use of bootstrapping and random feature selection techniques, and as such, it reduces variance and improves generalization capabilities.

The use of Random Forest models for volatility forecasting has become widespread recently due to complexity and nonlinearity of relationships between many macroeconomic and financial variables present in the financial market. Random Forest models can capture all those interactions without imposing any restrictions on the functional form and probability distribution. Another benefit of using Random Forest models lies in their ability to estimate the importance of input variables, which increases interpretability of the forecasts as compared with some other machine learning models. Finally, Random Forest models show good resistance to noise and overfitting and can handle high-dimensional financial data efficiently. However, Random Forest models often require significant computational power and are not always capable of capturing temporal dynamics present in the financial time series. Variable importance measures increase interpretability but do not make forecasting process easier as compared with conventional econometric models.

Extreme Gradient Boosting (XGBoost)

One of the most effective algorithms used currently in predictive analytics can be mentioned

as Extreme Gradient Boosting (XGBoost). XGBoost is the algorithm proposed by Chen and Guestrin (2016) and includes iterative training of decision trees along with gradient-based optimization technique aimed at improvement of prediction accuracy.

There are several advantages of XGBoost which allow it to be successfully used in finance. First, XGBoost takes into account the interactions and nonlinear relationships between variables as well as regularization to avoid the problem of overfitting. Secondly, XGBoost can operate with high-dimensional datasets and manage the missing values. Thus, it is especially suitable for forecasting in finance.

It should be noted that the number of studies proves that XGBoost usually provides higher prediction accuracy in comparison with the traditional econometric models and other machine learning algorithms while forecasting financial market volatility and returns (Gu et al., 2020). Therefore, XGBoost is widely applied in portfolio optimization, risk management, credit scoring, and algorithmic trading.

Still, there are some disadvantages of XGBoost which should be considered although this algorithm demonstrates very high prediction accuracy. First, it is quite difficult to configure properly the hyperparameters of XGBoost. Moreover, it is expensive to use XGBoost because of huge amount of data and complicated model configuration. Feature importance allows increasing the interpretability of XGBoost, but it is still much less interpretable than the traditional econometric models.

In conclusion, machine learning algorithms have enriched the set of methods for volatility forecasting and increased opportunities due to the ability to consider the nonlinear and high-dimensional nature of financial markets. At the same time, inability of these algorithms to deal with the sequential dependence of financial time series has brought about the emergence of deep learning approaches.

Deep Learning Models

The considerable improvement in computational power, development in the area of artificial intelligence and big data analysis have increased significantly the usage of the deep learning models in the volatility forecasting literature. The deep learning models denote the more advanced class of the machine learning algorithms that employ multi-layered nonlinear transformations to extract the hierarchical structure of the features from the data (Goodfellow, Bengio, & Courville, 2016). The difference between the traditional machine learning algorithms and deep learning models is the fact that in the second case complex features can be learned directly from the data without the need to engineer these features manually.

The applicability of the deep learning models in the financial forecasting problem was determined by the properties of the financial market as complicated and dynamic system characterized by the presence of temporal dependencies, regime changes and complicated interaction of the economic and financial variables. The complexity of the properties of the financial markets creates problems in the application of the traditional econometric and machine learning models, especially in the case of high dimensional data and rapidly changing market environment. Deep learning models solve these problems because they allow learning complex nonlinear dependencies and long-term dependencies that cannot be learned by other approaches.

The recent empirical research demonstrates that the deep learning models exhibit better forecasting ability compared to the econometric models and other machine learning models, especially in the case of the high-frequency financial data and complicated large data sets

(Fischer & Krauss, 2018). Therefore, the usage of the deep learning models in the volatility forecasting, asset pricing, algorithmic trading, portfolio optimization and financial risk management became widespread.

Nevertheless, despite the high prediction ability, the deep learning models have several disadvantages. First, the training of deep learning models is relatively expensive due to the usage of large amounts of data and strong computational power. Second, the deep learning models are black boxes; therefore, they do not provide any insight into the economic mechanism of predictions.

Long Short-Term Memory Networks (LSTM)

Long Short-Term Memory (LSTM) network architecture, introduced by Hochreiter and Schmidhuber (1997), is currently one of the most popular deep learning methods used in finance. LSTM networks are an instance of Recurrent Neural Networks (RNNs) that are specifically designed to learn long-term dependencies between sequential data elements.

The main reason for the design of the LSTM networks is associated with the necessity to solve the problem of vanishing gradients, which occurs commonly in standard recurrent neural networks. Due to the use of memory cells and gate mechanisms, LSTM networks can learn how to keep, modify, and forget the data depending on its importance. Such capability makes LSTM networks appropriate for the analysis of financial data because previous events often affect future values.

Nowadays, the applications of LSTM models to volatility forecasting are actively developing. Several empirical studies have shown that LSTM networks are able to forecast better than econometric models and some other machine learning algorithms in terms of volatility and returns of financial assets, especially if such relationships are non-linear and depend on time (Fischer & Krauss, 2018). Therefore, LSTM models are widely used in stock market prediction, risk management, derivatives pricing, and dynamic asset allocation.

However, there are some disadvantages of LSTM networks. First, the training of LSTM networks is computationally expensive and requires a lot of data. Moreover, although LSTM networks have excellent forecasting abilities, their complicated structure limits their interpretability and economic reasoning.

Gated Recurrent Units (GRU)

The Gated Recurrent Units (GRUs) introduced by Cho et al. (2014) can be regarded as a more simplified version of the LSTM network. Similarly to other LSTM models, the GRUs can learn sequential dependencies and temporal relations in the input values. The GRU networks, however, use a smaller number of gates and parameters, which makes them computationally efficient and enables quicker training.

The major advantage of the GRU models is the ability to ensure predictive accuracy like LSTM neural networks but consuming less computing power. Many research papers suggest that the GRUs have demonstrated their ability to model financial time series and to provide appropriate forecasting for the returns and volatility, particularly in cases where the computational complexity plays an important role.

Like any other model of prediction, GRUs suffer from the issues of interpretability and optimization of the model parameters. Moreover, the forecasting accuracy of GRU models can differ depending on the financial market and the forecast period. Therefore, the efficiency of GRU and LSTM models can strongly depend on the characteristics of the data set.

Transformer Models

The development of Transformer architecture has become one of the most notable achievements in artificial intelligence and sequence modeling recently. The Transformer model proposed by Vaswani et al. (2017) utilizes self-attention mechanism which allows for the simultaneous analysis of sequences and finding complex dependencies between them.

Transformer models have received considerable attention due to the use of such models for forecasting financial markets since financial systems have nonlinear relationships and dependencies between observations which cannot be easily captured through recurrent methods. Self-attention mechanisms allow for focusing on certain aspects from different points in time and analyzing dependencies over a long forecasting horizon.

According to recent research, Transformer-based architecture can perform significantly better than recurrent neural networks and some machine learning algorithms in forecasting financial time series and in capturing market volatility, especially in case of high-volume and high-frequency data sets (Lim & Zohren, 2021). As a result, Transformer-based models have gained much popularity in asset pricing, algorithmic trading, and portfolio management tasks.

Despite successful performance of Transformer models, such models are quite computationally expensive and usually require a lot of training data. Furthermore, like other deep learning models, Transformer-based models lack interpretability and face challenges in explaining the economic reason behind their forecasts. However, the possibility of capturing long-range dependencies makes Transformer architecture very promising in volatility forecasting and finance-related applications.

Hybrid Volatility Forecasting Models

Regardless of significant advances attained through econometric, machine learning, and deep learning methods, there is still no single forecasting approach capable of demonstrating consistently superior results in various types of financial markets and economic situations. Financial time series are rather complex and demonstrate the existence of linear and nonlinear interrelations, volatility persistence, asymmetry, long memory properties, and structural changes (Poon & Granger, 2003). In this situation, each forecasting approach does not provide sufficient consideration of all aspects of financial market dynamics. Such limitations of independent forecasting approaches have led to the creation of hybrid volatility forecasting frameworks aimed at uniting the advantages of several different approaches to overcome their limitations. Hybrid models rely on the assumption that different forecasting techniques capture different aspects of financial time series and thus allow predicting market risks better (Zhang, 2003).

Hybrid models in financial applications usually combine econometric approaches with machine learning or deep learning approaches. Econometric models like GARCH models and their extensions are rather efficient in volatility persistence, clustering, and conditional heteroskedasticity modeling, whereas machine learning and deep learning techniques capture nonlinear interrelationships, high dimensionality, and complex temporal dependencies. Thus, hybrid modeling approaches enable the simultaneous capturing of statistical and nonlinear properties of financial markets.

Some of the most used hybrid approaches include GARCH-LSTM frameworks where GARCH models are employed to model volatility dynamics, whereas LSTM networks are used to estimate nonlinearity and long-term dependencies. GARCH-XGBoost frameworks unite econometric volatility estimates with gradient boosting models to improve accuracy of the forecasts and modeling of complex interrelationships between financial and

macroeconomic variables. Some other hybrid frameworks like Wavelet-LSTM and attention-based deep learning models also allow making promising forecasts by splitting financial time series into different frequency components and identifying corresponding temporal dependencies.

Numerous empirical studies provide increasing evidence on the superiority of hybrid forecasting frameworks over independent econometric and machine learning models in forecasting financial market volatility during periods characterized by uncertainty, financial crises, and rapid changes in market dynamics (Kim & Won, 2018). Many research works demonstrate improved forecast accuracy, risk estimates, and value-at-risk calculation using hybrid modeling approaches. Moreover, more accurate volatility forecasts provided by hybrid models help to considerably increase the effectiveness of portfolio construction, dynamic asset allocation, and financial risk management decision-making.

However, hybrid forecasting frameworks also have certain limitations. The use of hybrid approaches significantly complicates models due to their increasing complexity and computational needs. In addition, hybrid frameworks are usually characterized by increased need for parameter tuning and large datasets. The black-box nature of such frameworks may decrease model interpretability and complicate economic interpretation of results obtained. Therefore, even though hybrid models are characterized by substantial predictive power, the balance between forecasting accuracy, computational efficiency, and interpretability remains one of the main challenges for future research.

Thus, hybrid forecasting frameworks constitute a rather important area of modern volatility forecasting research. Using the advantages of econometric models and artificial intelligence techniques, hybrid approaches enable considerable improvements in volatility predictions and help to make better investment and financial risk management decisions. Table II provides a comparative overview of the major classes of volatility forecasting methodologies.

TABLE II: Comparison of Major Volatility Forecasting Models

Model	Volatility Clustering	Asymmetry	Long Memory	Nonlinearity	Interpretability
Historical Volatility	No	No	No	No	High
EWMA	Partial	No	No	No	High
ARCH	Yes	No	No	No	High
GARCH	Yes	No	Partial	No	High
EGARCH	Yes	Yes	Partial	No	High
TGARCH	Yes	Yes	Partial	No	High
FIGARCH	Yes	Partial	Yes	No	Moderate
DCC-GARCH	Yes	Partial	Partial	No	Moderate
Machine Learning Models	Partial	Partial	Partial	Yes	Low
Deep Learning Models	Yes	Yes	Yes	Yes	Low
Hybrid Models	Yes	Yes	Yes	Yes	Moderate

IMPLICATIONS FOR PORTFOLIO OPTIMIZATION

In the age of portfolio management, volatility prediction is an essential process because volatility represents one of the major elements of investment risk and influences the process of portfolio construction, asset allocation, and risk management. The idea of portfolio optimization was first formulated by Markowitz (1952) and based on the assumption that investors attempt to optimize the relationship between risk and return. Therefore, efficient estimation of future volatility is crucial for evaluation of risk-return tradeoffs for securities and creation of optimal portfolios.

Time variation of risk, correlations, and periods of uncertainty in financial markets create a highly volatile environment where sudden changes in volatility can affect asset risk profiles and make static investment strategies inefficient. In conditions like that, underestimation of future volatility levels can lead to inefficient diversification, suboptimal asset allocation, and losses of portfolio value. On the contrary, proper volatility prediction allows investors to adapt to the changes in market conditions and use more sophisticated strategies (Poon & Granger, 2003). Thus, advancements in volatility forecasting methodology allow estimating risks better and making more efficient investments decisions. Today, econometrics, machine learning, deep learning, and hybrid forecasting approaches are increasingly used in portfolio management processes for better estimation of future uncertainty. Implications of volatility forecasting for investment decisions will be discussed further.

Volatility Forecasting and Portfolio Risk Estimation

Volatility forecasting also plays an instrumental role in the estimation of portfolio risk. Indeed, portfolio risk is primarily defined by asset variances and the correlation between assets. Given the time-varying nature of both variances and correlations, an accurate forecast of future volatility is needed to assess portfolio risk in a meaningful way.

In many cases, classical approaches make use of static assumptions regarding variances and correlation. At the same time, financial markets go through periods when uncertainties are high, and, consequently, risk characteristics vary greatly (Cont, 2001). Thus, static estimates of portfolio risks often do not reflect the real degree of exposure investors have during financial crises and other situations when risks increase drastically. With the help of forecasts that provide forward-looking measures of uncertainty, investors can calculate a more realistic measure of their future risk exposure.

The importance of accurate volatility forecasts in stress testing and scenario analysis should also be noted. Financial institutions increasingly apply advanced volatility models to analyze portfolio behavior under stressful market circumstances and to detect possible vulnerabilities in case of market shocks.

Mean-Variance Portfolio Optimization

Mean-Variance Optimization approach suggested by Markowitz (1952) is one of the most important concepts of modern portfolio theory. The definition of the concept of Mean-Variance Optimization may be formulated as follows: construction of an optimal portfolio of securities with the maximal return for the given risk or with the minimal risk for the given return. Since the concept of Mean-Variance Optimization uses portfolio variance as the measure of risk, the accuracy of estimations of volatility is very important when constructing an optimal portfolio.

The effectiveness of the Mean-Variance Optimization approach highly depends on the accuracy of estimations of variances-covariances. The errors in forecast of volatility may substantially reduce the efficiency of the portfolio weights and thus reduce the efficiency of

asset allocation and inefficiency of diversification. In case of overestimating volatility, the investors will be too conservative, and in case of underestimating volatility, the investors will take too much risk.

The invention of new approaches to volatility forecasting provided investors with a great number of opportunities for using Mean-Variance Optimization concept in practice. The estimations of dynamic volatility and correlation will help investors adjust the weights of portfolio according to changing volatilities and correlations.

Minimum Variance Portfolios

Minimum Variance Portfolios (MVPs) is a particular type of portfolio optimization techniques which emphasize the reduction of portfolio risk without considering the returns of the portfolio. The popularity of MVPs became rather high in recent years owing to the difficulty in estimating the expected return and risks resulting from the error in forecast of expected return.

Because MVPs rely heavily on the variance and covariance forecasts, improvement in volatility forecasting becomes highly important for the effectiveness of these types of techniques. Portfolio diversification and risk reduction can be done through better forecasts of the volatility. Thus, more advanced econometrics and machine learning models are frequently applied in order to improve volatility forecasting.

It has been seen in empirical research that portfolios constructed with the help of better forecasts of volatility tend to show superior performance when it comes to risk adjustment, compared to portfolios constructed with the assumption of static variances (DeMiguel, Garlappi, & Uppal, 2009). Hence, volatility forecasting has become highly important for minimum variance investments.

Risk Parity Strategies

In this manner, risk parity can be viewed yet another approach to managing investment portfolios by allocating investments proportionate to their risks with respect to the portfolio, not their returns or prices on the markets. With the use of risk parity, the principal goal is the allocation of risks of portfolio elements with an aim to achieve maximum diversification of the portfolio. The portfolio construction process regarding the use of risk parity approach is based exclusively on the assessment of asset volatilities and correlations. Due to variations in volatilities at different times, the application of static risk assessment methods can lead to uneven risk allocation and consequently inefficient portfolios. Risk forecasting makes it possible to adjust portfolio weights and risks under any market circumstances.

Due to the progress in the development of volatility forecasting techniques in recent years, it has become much simpler to implement risk parity approaches. Prediction of market uncertainties plays a crucial role in risk allocation and reacting to stressful situations.

Dynamic Asset Allocation and Portfolio Rebalancing

The main implication that arises regarding volatility forecasting pertains to its application in dynamic asset allocation and portfolio rebalancing. In contrast to traditional static approach in investment, the dynamic approach makes it possible to account for the fact that financial markets are constantly experiencing fluctuations in risk levels and therefore portfolio weights change accordingly.

The data about the expected fluctuations in market uncertainty, gained thanks to volatility forecasting, can be applied in dynamic asset allocation. A high level of volatility serves as a sign that suggests decreasing the number of risky investments and increasing the number of

defensive investments. In case when the level of uncertainty on the market is relatively low, then it becomes possible to increase risky investments. Modern developments in forecasting in econometrics and machine learning provide investors with an opportunity to apply adaptive investment strategies.

Risk Management Applications

Besides portfolio management and asset allocation, there exist numerous significant applications of volatility forecasting in the risk management of financial institutions. Specifically, the forecasted volatilities can be applied for calculating important risk measures like VaR, ES, capital adequacy requirements, and stress tests. Thus, from this perspective, the accuracy of volatilities forecasts affects the accuracy of the risk management process.

Moreover, the accurate volatilities forecasts can be employed for pricing derivatives, managing risks of investments and analyzing the risks exposure of the investors. Finally, currently the regulators and policy makers use volatility forecasts for identifying the weak spots of the financial systems and for estimating the stability of the financial institutions.

In conclusion, the applications of volatility forecasts indicate that their usage is not limited to volatility forecasting only. Instead, volatility forecasting is a very important element of financial management and decision making. Therefore, the development of volatility forecasting is extremely important for effective portfolio management and risk assessment.

Table III summarizes the major applications of volatility forecasting in portfolio optimization and risk management.

TABLE III: Applications of Volatility Forecasting in Portfolio Optimization

Application Area	Role of Volatility Forecasting	Expected Benefits
Portfolio Risk Estimation	Provides forward-looking estimates of variances and correlations.	Improved assessment of portfolio risk exposure.
Mean-Variance Optimization	Enhances estimation of covariance matrices and portfolio weights.	More efficient risk-return trade-offs.
Minimum Variance Portfolios	Improves estimation of conditional variances and correlations.	Lower portfolio risk and improved diversification.
Risk Parity Strategies	Facilitates dynamic estimation of asset risk contributions.	Balanced portfolio risk allocation.
Dynamic Asset Allocation	Supports tactical rebalancing decisions under changing market conditions.	Enhanced adaptability and resilience.
Risk Management	Improves Value-at-Risk (VaR), stress testing, and hedging strategies.	More effective financial risk management and capital allocation.

RESEARCH GAPS AND FUTURE RESEARCH AGENDA

Although there have been substantial improvements in forecasting models of volatility, many issues still exist from both a theoretical and practical perspective. The present literature has made great strides in our understanding of financial markets and has produced some complex methods of forecasting volatility. However, the complex and changing nature of financial markets continues to reveal certain flaws in the existing methodology. It is critical to recognize these flaws to provide a roadmap for further research.

Major Research Gaps

Despite the rich body of literature related to the topic, there are no universally superior forecasting methods. In many cases, the quality of forecasting produced by various methodologies differs significantly depending on the type of market, the asset class, the forecast horizon, and economic environment. Those models that work well in one specific environment have much lower predictive power in other circumstances (Poon & Granger, 2003).

Secondly, many forecasting models currently available cannot simultaneously capture the variety of stylized facts that are observed in financial time series. Econometric models perform very well in capturing volatility clustering and persistence but have problems with accounting for the non-linear dependencies and breaks in time series (Gu, Kelly, & Xiu, 2020). On the contrary, machine learning and deep learning techniques have proven to be strong predictors but often lack interpretability of how the economic process behind the prediction works.

The third limitation that needs to be considered relates to the low attention paid to the robustness of forecasting models at times of financial crises and increased uncertainty. The regime of financial markets often changes dramatically due to economic, geopolitical, pandemic or any other unforeseen shock. The assumption of relative stability of the relation between variables used for forecasting often makes those models inefficient during turbulent times (Cont, 2001).

Moreover, despite the abundance of papers on statistical forecasting accuracy, the economic value of volatility forecasts receives comparatively little attention. Improvements in forecasting precision do not always result in better portfolio management and portfolio diversification. Additional research is needed to evaluate the forecasting models considering both statistical and economic performances of forecasting methods.

Finally, there are comparatively few works devoted to the application of alternative data and macroeconomic uncertainty indices in constructing forecasting systems. The growth of alternative data sources such as textual information, financial news, social media sentiment, high-frequency data, etc., opens the door for further research.

Explainable Artificial Intelligence in Volatility Forecasting

Despite the impressive ability of machine learning and deep learning models to generate accurate predictions, the use of these models in finance is hampered by their lack of interpretability. Most artificial intelligence tools work as black boxes and do not offer enough insight into economic forces behind their forecasts.

With a rising focus on the need for transparency and accountability in finance, there is a greater demand for the development of explainable artificial intelligence (XAI). In the future, researchers need to explore how explainability methods like SHapley Additive exPlanations (SHAP), Local Interpretable Model-Agnostic Explanations (LIME), and attention-based explanations can be applied to enhance the interpretability of volatility forecasting models.

Hybrid Econometric-Machine Learning Frameworks

However, the limitations of the used individual methods of forecast can give the grounds for developing an idea of hybrid forecasting models. Econometrics is highly effective in measuring persistence and conditional heteroskedasticity, while machine learning and deep learning methods can be successfully applied to modeling non-linear relationships.

Thus, future research might benefit from developing an integrated model combining the

advantages of two mentioned approaches. This will provide an opportunity to obtain more accurate uncertainty measures using hybrid forecasting models, such as GARCH, deep learning and ensemble models.

Regime-Dependent Forecasting Models

There is always a chance that the financial markets would transition from the stable regime to the highly unstable regime. Changes are expected in the relationship between the financial variables in relation to the regime switch, and the reliability of predictions made through conventional models might be affected.

It would be essential to conduct research aimed at devising forecasting models which would take into consideration the regime switches. The models that could be considered in this context include Markov switching models, attention based neural networks, and adaptive learning models.

Alternative Data and Real-Time Forecasting

Rapid growth in the application of information technology and digital systems has resulted in an unprecedentedly high amount of alternative data in finance. Data acquired from financial news, social networks, web searches, and textual data affects the sentiment of investors and their expectations about future market performance.

Further research should be devoted to the development of techniques aimed at incorporating the alternative data in volatility forecasts. Natural language processing, sentiment analysis, and real-time data analytics can provide significant improvements in forecasting accuracy and timeliness of market risk assessments.

Uncertainty-Aware Portfolio Optimization

In view of the growing trends of financial crisis, geopolitical tension and macroeconomic disturbance, it has become important for scholars to come up with models of portfolio optimization that will take into consideration the problem of uncertainties in the financial markets. Usually, most of the models do not take into consideration any risk or correlation change, hence the models become useless when utilized in stressful market conditions.

In the future, scholars should focus on coming up with portfolio optimization models that will consider the uncertainties and combine aspects of the volatility forecasting model and robust optimization. These models will make portfolios more resilient, provide better diversification and facilitate investment decision making.

Overall, in the future, studies on volatility forecasting should take into consideration the utilization of econometrics, artificial intelligence and non-traditional data as well as uncertain investment strategies. This will go a long way in not only enhancing knowledge about financial market volatilities but also coming up with new forecasting systems for portfolio optimization and financial risk management.

CONCLUSION AND POLICY IMPLICATIONS

CONCLUSION

Volatility is regarded as one of the key variables, which serve as the predictor of investment risks and play an important role in pricing and portfolio management. In terms of the increased frequency of financial crisis, geopolitical events, uncertainty of policies, and interconnectedness of markets, financial systems become more complex, requiring advanced methods of forecasting. Thus, volatility forecasting becomes an important issue for research in finance as well as portfolio management and risk measurement practice.

The current literature review provides a thorough analysis of the historical development of various methodologies of volatility forecasting, including statistical methods, econometrics, machine learning, deep learning, and hybrid approaches. Historical Volatility and Exponentially Weighted Moving Average methods are easy to calculate; however, they fail to capture dynamics of financial markets. Development of econometrics has introduced such popular family of models as ARCH and GARCH, which is a breakthrough in modeling of the volatility as a time-varying stochastic process characterized by persistence and clustering. Developments in the field of machine learning and deep learning have enabled modeling of nonlinear processes, interactions, and long-run dependencies and provided an arsenal of tools for forecasting of volatility. Moreover, hybrid approaches enable combination of the benefits of both econometrics and artificial intelligence.

Apart from discussing volatility forecasting methodologies, the review analyzes their implication for portfolio optimization and financial risk management practice. Reliability of volatility forecasts allows better assessment of portfolio risk, calculation of covariances between assets, and allocation of assets. Better forecasting models enhance practical use of mean variance optimization, minimum variance portfolios, risk parity strategy, and dynamic asset allocation models. Volatility forecasting is used also for pricing derivatives, calculation of the Value at Risk, stress testing and other risk management practices. Therefore, improvements in forecasting models provide implications beyond statistical forecasting and impact investment performance.

To conclude, the results of the review suggest that volatility forecasting is an essential tool for finance. With increasing complexity of financial systems, it becomes necessary to develop more accurate and flexible methodologies to improve theory of portfolio optimization and practice of financial risk management.

POLICY IMPLICATIONS

Several major implications for investors, financial institutions, and regulators follow from the results of this review. First, financial institutions should consider implementing advanced volatility forecasting systems combining econometrics, machine learning, and deep learning methods. In case financial institutions will be relying on simple historical estimates of the level of riskiness, they may underestimate actual market uncertainty, especially under financially stressful periods. Thus, the implementation of the mentioned advanced volatility forecasting framework should become a part of the risk management system.

Second, institutional investors and portfolio managers should use dynamic asset allocation strategies that are based on the use of volatility forecasts. As market risks and correlations between assets vary greatly over time, static investment strategies may become inefficient under dynamically changing conditions. Advanced forecasts of market volatilities can help to better allocate investments and diversify portfolios.

Third, regulators should update stress testing and financial surveillance practices by using advanced methodologies for forecasting volatility. In the light of the increasing frequency of the financial crises and contagion processes, continuous monitoring of dynamically changing risk situation is becoming very important. Advanced volatility forecasting systems may help to detect risks and take actions to maintain financial stability.

Finally, policymakers should promote the development of explainable artificial intelligence frameworks and investments in financial data infrastructure. Alternative data and high-frequency data can significantly increase forecast accuracy. Simultaneously, improving model transparency can help to foster investments in artificial intelligence tools.

Limitations and Future Research

However, even though the provided literature review includes an extensive synthesis of the literature concerning the issue of forecasting volatility and its practical significance for portfolio optimization, there are some gaps in the presented study. Firstly, the reviewed literature is of conceptual character and there is no practical comparison between different forecasting methodologies applied to specific sets of data and financial environments. Thus, the relative forecasting power of various models differs across various financial markets, assets and horizons.

Secondly, the fast development of artificial intelligence and financial technologies implies the development of many new forecasting methodologies which have not been considered in the current literature review.

Thirdly, the review pays attention to the practical significance of the problem of forecasting but does not consider the question of estimation of the economic value that can be obtained through the improvements in the forecasting performance of the model. The improvements in forecast performance do not imply better results in portfolio management.

The following issues can be considered in future research on the topic of volatility forecasting. Firstly, future studies should focus on the development of frameworks based on the approaches to explainable artificial intelligence that would imply consideration of both forecasting performance and economic interpretation. Econometric methods should be considered together with machine learning and deep learning methods.

Moreover, future research should pay attention to the use of alternative information sources such as financial news and market data to increase the reaction of volatility forecast to the changes in the market environment.

Another direction of further research concerns the development of the framework of regime-dependent and uncertainty-aware forecasting. Additional research is necessary for studying the implementation of advanced forecasting methods in the portfolio optimization framework and evaluation of the effect of the latter on the diversity and risk-adjusted performance of portfolios.

Thus, it is possible to state that the future of the methodology of volatility forecasting relates to the combination of econometric methods, artificial intelligence methods and dynamic portfolio management. The solution to the issues would be valuable for the development of financial forecasting theories and investment and risk management practices in the global financial market.

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